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GAMS Model

The Executive Reader

GAMS optimization is used to determine the optimal shipping of gas from any set of exporting countries to any set of importing countries using any set of methods of transportation. Costs of building, expansion, and operation are incorporated into the model. GAMS models the parameters to determine the highest net present value out of all possibilities. There are 14 exporting countries and 10 importing countries and 5 methods of transportation to consider. If this were to be done by hand there would be $14^{10}*14!*5 = 1.26*10^{23}$ possibilities in choice alone. This doesn't take into account varying the amount shipped and expansion possible for each year over the life of the project. Using GAMS eliminates many options and simplifies the model to determine the optimum value in a short time.

The Detailed Reader

Variables are initialized that are used to vary the optimal transportation for the model.

Variables

x(i,j,m,t)	shipment of natural gas
c1(i,j,m)	capacity at the exporting plant
c2(j,m,t)	capacity at the importing plant
e1(i,m,t)	expansion of the exporting plant
e2(j,m,t)	expansion at the importing plant
invest1(i,m,t)	investment of exporting plant
invest2(j,m,t)	investment of importing plant
invest3(i,j,m,t)	investment of just laying the pipe
z(i,j,m,t)	pipe binary value set by y1 and y2
NPV	net present value
y1(i,m,t)	binary variable for the exporting plant
y2(j,m,t)	binary variable for the importing plant

The all of the variables except the binary variables are set as positive because a negative value any of these could possibly yield a higher NPV, plus negative values for these variables is not possible for the process.

Parameters are set for the equations that are used to model the project. The following parameters are used in the equations that are later explained.

fci1(i,m) fixed capital investment for the exporting plant

fci1(i,m) = explant(i,m) + exhold(i,m)

fci2(j,m) fixed capital investment for the importing plant
fci2(j,m) = implant(j,m) + imhold(j,m)

capital1(i,m) cost for the exporting country to change NG to a method
capital1(i,m) = explantper(i,m) + exholdper(i,m)

capital2(j,m) cost for the importing country to change a method to NG
capital2(j,m) = implantper(j,m) + imholdper(j,m)

opex(i,m) cost to operate at the exporting country
opex(i,m) = opexplant(i,m) + opexhold(i,m)

opim(j,m) cost to operate at the importing country
opim(j,m) = opimplant(j,m) + opimhold(j,m)

opship(i,j,m) cost to operate shipping method
opship(i,j,m) = dist(i,j)*distcost(m)

The equations in GAMS are defined in the following list

Equations list

exportLim(i,m,t)	can't exceed the exporting plant limit
importLim(j,m,t)	can't exceed the importing plant limit
totalship(j,t)	can't exceed the demand at importing country
exportExp(i,m,t)	if no plant built at exporting country then there is no expansion at exporting country
importExp(j,m,t)	if no plant built at importing country then there is no expansion at importing country
totalexcap(i,m,t)	total exporting plant capacity
totalimcap(j,m,t)	total importing plant capacity
investEx(i,m,t)	investment at exporting country
investIm(j,m,t)	investment at importing country
investpipe(i,j,m,t)	investment of pipe
expipe(i,j,m,t)	no export plant no pipe
impipe(i,j,m,t)	no import plant no pipe
eximpipe(i,j,m,t)	no export or import plant no pipe
budgetLim(t)	can't exceed the budget
NetPresentValue	Net Present Value

The variables and parameters that are used for each of the above equations are listed below with a short description of what is taking place and why the equation is used. The actual code isn't listed, but an easier to read form is used. The actual code is written out later.

Equations listed in easy to read format

exportLim(i,m,t) .. $\sum_{j} x(i, j, m, t) \le c1(i, m, t)$ This equation is to limit the total amount shipped to the importing countries to not exceed the capacity of an exporting plant.

importLim(j,m,t) .. $\sum_{i} x(i, j, m, t) \le c2(j, m, t)$ This equation is to limit the total amount shipped to an importing county to not exceed the capacity of the importing country.

totalship(j,t) .. $\sum_{i}\sum_{m} x(i, j, m, t) \le demand(t, j)$ This equation limits the total amount shipped to an importing country of all methods used to not exceed the demand at the importing country at a particular time.

exportExp(i,m,t) .. e1(i,m,t) ≤ y1(i,m,t)*10^7 There can't be expansion at an exporting plant without an exporting plant. This is done by setting the expansion less than a binary variable times a maximum expansion.

importExp(j,m,t) .. e2(j,m,t) ≤ y2(j,m,t)*10^7 There can't be expansion at an importing plant without an importing plant. This is done by setting the expansion less than a binary variable times a maximum expansion.

totalexcap(i,m,t) .. c1(i,m,t) = c1(i,m,t-1) (if t>1) + e1(i,m,t) The total expansion at an exporting plant up to the current year. There is no capacity in the first year so the first year is just set to the expansion of the first year by the if statement.

totalimcap(j,m,t) .. c2(j,m,t) = c2(j,m,t-1) (if t>1) + e2(j,m,t) The total expansion at an importing plant up to the current year. There is no capacity in the first year so the first year is just set to the expansion of the first year by the if statement.

investEx(i,m,t) .. invest1(i,m,t) = fci1(i,m)*y1(i,m,t) + capital1(i,m)*e1(i,m,t) This is the investment at the exporting plant. The fci is set to the fixed capital investment for a method at an exporting country. This is multiplied by the binary variable for optimization of the model. The capital cost of expanding the plant times the expansion is then added to give the investment cost.

investIm(j,m,t) .. invest2(j,m,t) =e= fci2(j,m)*y2(j,m,t) + capital2(j,m)*e2(j,m,t) This is the investment at the importing plant. The fci is set to the fixed capital investment for a method at an importing country. This is multiplied by the binary variable for optimization of the model. The capital cost of expanding the plant times the expansion is then added to give the investment cost.

investpipe(i,j,m,t) (if m=pipe) .. invest3(i,j,m,t) = dist(i,j)*pipecost(m)*z(i,j,m,t) The cost of installing the pipe from the selected countries is treated as a separate cost. The investment equals the distanace between the countries times the cost per mile of the pipe. z is related to the binary variables y1 and y2.

expipe(i,j,m,t) (if m=pipe) .. z(i,j,m,t) $\leq$ y1(i,m,t) There is no pipe built if there isn't an exporting plant

impipe(i,j,m,t) (if m=pipe) .. z(i,j,m,t) $\leq$ y2(j,m,t) There is no pipe built if there isn't an importing plant

eximpipe(i,j,m,t) (if m=pipe) .. z(i,j,m,t) $\geq$ y1(i,m,t) + y2(j,m,t) –1 There is no pipe built if there is no exporting or importing plant

budgetLim(t) ..

$$\sum_i \sum_m invest1(i,m,t) + \sum_j \sum_m invest2(j,m,t) + \sum_i \sum_j \sum_m invest3(i,j,m,t)(if\ m = pipe) \leq budget(t)$$

The summation of all investments made can not exceed the budget at a specific time.

NetPresentValue ..

$$NPV = \sum_t \Bigg\{ \Bigg[\sum_i \sum_j \sum_m \big(price(t) * x(i,j,m,t) - \big(opex(i,m) + opim(j,m) + opship(i,j,m) \big) * x(i,j,m,t) \big)$$
$$- \sum_i \sum_m invest1(i,m,t) - \sum_j \sum_m invest2(j,m,t) - \sum_i \sum_j \sum_m invest3(i,j,m,t)(if\ m = pipe) \Bigg] * dividend(t) \Bigg\}$$

The price that the gas is sold for times the amount shipped minus the total operating costs times the amount shipped minus the total investment costs with all of this multiplied by the inverse of the quantity of one plus the interest rate with the quantity raised to the year is equal to the net present value. Net present value is simply the revenue at each year divided by the interest rate as in the following equation.

$$NPV = \sum_t \frac{revenue\ at\ year\ t}{(1+i)^{year}}$$

The code of the above equations that is used in GAMS is listed below.

```
exportLim(i,m,t) ..      sum(j, x(i,j,m,t)) =l= c1(i,m,t);
importLim(j,m,t) ..      sum(i, x(i,j,m,t)) =l= c2(j,m,t);
totalship(j,t) ..        sum((i,m), x(i,j,m,t)) =l= demand(t,j);
exportExp(i,m,t) ..      e1(i,m,t) =l= y1(i,m,t)*maxexp;
importExp(j,m,t) ..      e2(j,m,t) =l= y2(j,m,t)*maxexp;
totalexcap(i,m,t) ..     c1(i,m,t) =e= c1(i,m,t-1)$(ORD(t)>1) + e1(i,m,t);
totalimcap(j,m,t) ..     c2(j,m,t) =e= c2(j,m,t-1)$(ORD(t)>1) + e2(j,m,t);
investEx(i,m,t) ..       invest1(i,m,t) =e= fci1(i,m)*y1(i,m,t) + capital1(i,m)*e1(i,m,t);
investIm(j,m,t) ..       invest2(j,m,t) =e= fci2(j,m)*y2(j,m,t) + capital2(j,m)*e2(j,m,t);
investpipe(i,j,m,t)$(ORD(m)=4) ..   invest3(i,j,m,t) =e= dist(i,j)*pipecost(m)*z(i,j,m,t);
expipe(i,j,m,t)$(ORD(m)=4) ..       z(i,j,m,t) =l= y1(i,m,t);
```

```
impipe(i,j,m,t)$(ORD(m)=4) ..       z(i,j,m,t) =l= y2(j,m,t);
eximpipe(i,j,m,t)$(ORD(m)=4) ..     z(i,j,m,t) =g= y1(i,m,t) + y2(j,m,t) -1;
budgetLim(t) ..          sum((i,m),invest1(i,m,t)) + sum((j,m),invest2(j,m,t)) +
sum((i,j,m)$(ORD(m)=4),invest3(i,j,m,t)) =l= budget(t);

NetPresentValue ..       NPV =e= sum(t,(sum((i,j,m),price(t)*x(i,j,m,t)-
(opex(i,m)+opim(j,m)+opship(i,j,m))*x(i,j,m,t))-sum((i,m),invest1(i,m,t))-
sum((j,m),invest2(j,m,t))-sum((i,j,m)$(ORD(m)=4),invest3(i,j,m,t)))/interest(t));
```

GAMS CODE:

```
set
        i Chemicals used /NG,LNG,ME, AM,FA /
        j Markets / USA, VZ, COL, TT /
        k Process / FAP , MEP,  LNGP, AMP /
        l Time /T1,T2/

        mn(k,i)
              /FAP.FA
               MEP.ME
               LNGP.LNG
               AMP.AM /
              ;
alias (i,ii),(j,jj),(k,kk);

parameter
        price(i) price of chemicals (dollar-ton)
               /NG     .27
               LNG     .47
               ME      20.61
               AM      225.97
               FA      800/


        alpha(k) variable investment cost  (dollar_capacity)
               /FAP    10608
               MEP     109.93
               LNGP    81.67
               AMP     152.82/


        beta(k) fixed investment cost (dollar)
               /FAP    700000000
               MEP     40000000
               LNGP    60000000
               AMP     40000000/
```

```
        delta(k) unit operating cost (dollar_capacity)
              /FAP   .8
              MEP    .8
              LNGP   .8
              AMP    .8 /

table    mu(i,k) positive constant characteristic of process
                    FAP     MEP     LNGP       AMP
              NG      0.32    0.41    1          0.62
              LNG     0       0       1          0
              ME      0.77    1       0          0
              AM      0       0       0          1
              FA      1       0       0          0
;
table
        upper_s(i,j) upper supply of chemicals  (metric ton_year)
                    USA        VZ        COL      TT
              NG      0          0          0        3352455
              LNG     0          0          0        0
              ME      6110000    6548000    25000    550000
              AM      0          0          0        0
              FA      0          0          0        0
;

table
        lower_s(i,j) lower supply of chemicals   (metric ton_year)
                    USA        VZ        COL       TT
              NG      0          0          0        0
              LNG     0          0          0        0
              ME      0          0          0        0
              AM      0          0          0        0
              FA      0          0          0        0


table
        upper_d(i,j) upper demand of chemicals   (metric ton_year)
                    USA        VZ         COL        TT
              NG      0          0          0          2771908
              LNG     71000000   3630000    599000     0
              ME      3400000    8800       140        600000
              AM      6240000    640000     70800      0
              FA      50000      0          0          0
;
```

```
 table
        tranport(i,j) transportation from Trinidad to Country X
                USA      VZ       COL     TT
        NG      0        0        0        0
        LNG     0.9      .97      .95      0
        ME      0.9      .97      .95      0
        AM      0.9      .97      .95      0
        FA      0.9      0        0        0

;
table
        lower_d(i,j) lower demand of chemicals   (metric ton_year)
                USA      VZ       COL     TT
        NG      0        0        0        0
        LNG     0        0        0        0
        ME      0        0        0        0
        AM      0        0        0        0
        FA      0        0        0        0
;

table    lower_c(k,l)    lower bounds for the capacity expansions
                T1    T2
        FAP     0     0
        MEP     0     0
        LNGP    0     0
        AMP     0     0
;

table    upper_c(k,l)    upper bounds for the capacity expansions
                T1      T2
        FAP     46000   55000
        MEP     550000  600000
        LNGP    4000000 5000000
        AMP     620000  800000
;

variables

        NP              net present value
        op_cost(l)      operating cost
        prof(l)         profit
        sales1(l)       Total sales (without transportation)
        sales_trans(l)  sales include transportation
        Q(k,l)          capacity
        DQ(k,l)         capacity  EXPANSION
        W(i,k,kk,l)     flowrate (mass)streams of plants
```

```
        P(i,j,k,l)      purchased chemicals
        S(i,j,k,l)      sold chemicals
        CI(l)           Capital investment
        prod_rate(k,l)  production rate
        y(k,l)
;
free variables

        NP              net present value
        OP_COST(l)      operating cost
        CI(l)           Capital investment
        prof(l)         profit
        sales1(l)       Total sales (without transportation)
        sales_trans(l)  sales include transportation
;
positive variables

        Q(k,l)          capacity
        DQ(k,l)         capacity  EXPANSION
        W(i,k,kk,l)     flowrate (mass)streams of plants
        P(i,j,k,l)      purchased chemicals
        S(i,j,k,l)      sold chemicals
        prod_rate(k,l)  production rate
;
binary variables

        y(k,l)
;

equations

        constr1(k,l)      capacity constraint upper
        constr3(k)        current capacity
        constr4(k,l)      current capacity
        prfap (l)
        prmep(l)
        pramp(l)
        prlngp(l)
        inst_cap(k,l)     installed capacity

        upp_supp1(l)
        upp_supp2(j,l)
        upp_demdfa(j,l)
        upp_demdme(j,l)
        upp_demdam(j,l)
        upp_demdlng(j,l)
```

```
        upp_demd3(k,j,l)
        comp_bal1lngp(l)
        comp_bal1amp(l)
        comp_bal1mep(l)
        comp_bal1fap1(l)
        comp_bal1fap2(l)

        sales_transp(l)        Sales include transportation cost
        sales(l)               Total sales (without transportation)
        Profit(l)              profit of time l
        capital_investment1(l)  Capital investment
*         capital_investment(l)   Avaibility Capital Investment
        fix_cost2(k,l)
        OPERATING(l)           operating cost
        net_present_value      net present value of the project
;


        constr1(k,l)..              DQ(k,l) =l= upper_c(k,l)*y(k,l);
        constr3(k)  ..              Q(k,"T1") =e= DQ(k,"T1");
        constr4(k,l)$(ord(l) ne 1)..      Q(k,l) =e= Q(k,l-1)+DQ(k,l);

        prfap(l)..  prod_rate('fap',l) =e= S('fa','usa','fap',l)+
                    S('fa','vz','fap',l)+S('fa','col','fap',l)+ S('fa','tt','fap',l);
        prmep(l)..  prod_rate('mep',l) =e= S('me','usa','mep',l)+
                    S('me','vz','mep',l)+S('me','col','mep',l)+ S('me','tt','mep',l);
        pramp(l)..  prod_rate('amp',l) =e= S('am','usa','amp',l)+
                    S('am','vz','amp',l)+S('am','col','amp',l)+ S('am','tt','amp',l);
        prlngp(l)..  prod_rate('lngp',l) =e= S('lng','usa','lngp',l)+
                    S('lng','vz','lngp',l)+S('lng','col','lngp',l)+ S('lng','tt','lngp',l);


        inst_cap(k,l)..          Q(k,l) =g= prod_rate(k,l);

        upp_supp1(l)..           upper_s('ng','tt') =g= sum(k,P('ng','tt',k,l));
        upp_supp2(j,l)..         upper_s('me',j) =g= P('me',j,'fap',l);


     upp_demdfa(j,l)..          upper_d('fa',j) =g=  S('fa',j,'fap',l);
     upp_demdme(j,l)..           upper_d('me',j) =g=  S('me',j,'mep',l);
     upp_demdam(j,l)..           upper_d('am',j) =g=  S('am',j,'amp',l);
     upp_demdlng(j,l)..          upper_d('lng',j) =g=  S('lng',j,'lngp',l);
     upp_demd3(k,j,l)      ..     S('ng',j,k,l)=e=0;

     comp_bal1lngp(l)    ..   p('ng','tt','lngp',l)=e= mu('ng','lngp') *prod_rate('lngp',l);
     comp_bal1amp(l)    ..    p('ng','tt','amp',l)=e= mu('ng','amp') *prod_rate('amp',l);
```

```
    comp_bal1mep(l)    ..    p('ng','tt','mep',l)=e= mu('ng','mep') * prod_rate('mep',l);
    comp_bal1fap1(l)    ..   p('ng','tt','fap',l)=e= mu('ng','fap') * prod_rate('fap',l);
    comp_bal1fap2(l)    ..   W('me','mep','fap',l)+ sum(j,p('me',j,'fap',l))
                           =e= mu('me','fap') * prod_rate('fap',l);


       Sales_transp(l)..       sales_trans(l) =e=
SUM(j,tranport('lng',j)*price('lng')*S('lng',j,'lngp',l))  +
                             SUM(j,tranport('am',j)*price('am')*S('am',j,'amp',l))  +
                             SUM(j,tranport('me',j)*price('me')*S('me',j,'mep',l))     +
                             SUM(j,tranport('fa',j)*price('fa')*S('fa',j,'fap',l))
;
       sales(l)..                sales1(l) =e=  SUM(j,price('lng')*S('lng',j,'lngp',l))  +
                               SUM(j,price('am')*S('am',j,'amp',l))  +
                                SUM(j,price('me')*S('me',j,'mep',l))     +
                               SUM(j,price('fa')*S('fa',j,'fap',l))
;
       Profit(l)..                 prof(l) =e=  sales_trans(l) -
                                SUM(k,price('ng')* p('ng','tt',k,l))   -
                                SUM(j,price('me')* p('me',j,'fap',l))
                                -SUM(k,delta(k)*prod_rate(k,l));

     capital_investment1(l)..      SUM((kk),alpha(kk)*Q(kk,l)+ beta(kk)*y(kk,l)) =e= CI(l)
;



     fix_cost2(k,l)..            upper_c(k,l) *y(k,l) =g=  Q(k,l)
;

     OPERATING(l)..              op_cost(l) =e= SUM((k),delta(k)*prod_rate(k,l))
;

     net_present_value..         NP =e= -SUM((k,l),alpha(k)*Q(k,l)+ beta(k)*y(k,l))
                                   + SUM(l,prof(l))

;

model

         Trinidad        /all/ ;
 trinidad.OPTFILE = 1;

solve

         Trinidad using mip MAXIMIXING NP ;
```

```
OPTION LIMROW = 100;

display  w.l, np.l, dq.l, q.l, p.l, s.l, y.l, prof.l,op_cost.l, prod_rate.l,sales_trans.l,sales1.l,
CI.l ;
```